QUASI-COHERENT IMAGERY

This memo describes the imaging process in an optical system when the light is monochromatic but spatially only quasi-coherent. The imaging process in one dimension will be considered first. The result will then be extended to two dimensions. The model (Fig. 1) will be characterized by the following:

1. The light is temporally coherent with wavelength, λ.
2. The polarization is constant over any transform plane.
3. The scale variables are normalized as follows:

   a. $$w = \left(\frac{\lambda F_1}{2\pi}\right) x_2' \tag{1}$$

   b. $$x = \left(\frac{F_1}{F_2}\right) x_3' \tag{2}$$

The primed quantities are real distances, in the second and third planes respectively. F_1 and F_2 are the focal lengths of the transform lenses.

4. The first plane is characterized by the electric field $u(x,t)$. The electric field is the only function of time, t. The electric field transmitted by a transparency is equal to the incident field multiplied by electric field transmission of the transparency, $a(x)$. The function $a(x)$ includes both attenuation and phase information. The light intensity transmission of the transparency is:

$$t_f(x) = |a(x)|^2 \tag{3}$$

5. The second plane will be characterized by a "spatial frequency filter" with a field transmission $f(w)$.

6. The third plane is the image plane which is characterized by the light intensity, $I(x)$, averaged over time so that $I(x)$ unlike the electric field is independent of time. The analysis is based upon the transmission of the electric field. Only the film in the image plane, which is an energy sensing device, depends upon the light intensity. The film responds to the light intensity integrated over the exposure time. The light intensity is the time average of the square of the modulus of the electric field incident upon the film.

The image light intensity can be written from Fourier transform theory.[1,2]

$$I(x) = \overline{C_1 \left| F(x) * [a(x) \cdot u(x,t)] \right|^2} \tag{4}$$

For convenience all equations have been written in terms of a single constant C_i, $i = 1, 2, \ldots$. The bar denotes time average and * convolution.

$$a(x) \longleftrightarrow A(w) \tag{5}$$

$$F(x) \longleftrightarrow f(w) \tag{6}$$

The double arrows in equations 5 and 6 signify Fourier transform pairs. The lower case letter is the actual transparency or filter.

The following equations may be obtained for the image intensity if the indicated operations of equation 4 are carried out.[2,3]

$$I(x) = \int I'(w', x) S_x(w') \, dw' \tag{7}$$

$$I'(w', x) = \left| \int f(w) A(w - w') e^{-jwx} \, dw \right|^2 \tag{8}$$

$$S_x(w') = C_2 \int \Gamma_{x,x+s}(0) \, e^{jw's} \, ds \tag{9}$$

$$\Gamma_{x,x+s}(0) = \overline{u(x,t) u^*(x+s,t)} \tag{10}$$

Equation 8 is the intensity if $A(w)$ is shifted by w' with respect to the spatial filter. Equation 9 represents the probability that the transform $A(w)$ is shifted by w'. The shifting is caused by the lack of spatial coherence of $u(x,t)$. Equation 10 is the mutual coherence function of the incident field at the two points x and $x+s$. The notation is taken from reference 1.

Equation 7 may be rewritten.[2,3]

$$I(x) = \int T(w_1, w_2) A(w_1) A^*(w_2) e^{j(w_1 - w_2)x} \, dw_1 \, dw_2 \tag{11}$$

$$T(w_1, w_2) = C_3 \int S_x(w') f(w_1 - w') f^*(w_2 - w') \, dw' \tag{12}$$

Equations 11 and 12 give the image intensity in terms of a single function, $T(w_1, w_2)$, which combines the effects of spatial frequency filtering and the degree of spatial coherence. $T(w_1, w_2)$ represents the joint probability that the two frequencies $w = w_1$ and $w = w_2$ pass through the spatial filter at the same instant of time. Equations 11 and 12 may be simplified for the familiar cases of complete coherence and incoherence.

Perfect Coherence

The mutual coherence function is unity for perfect coherence. Equation 9 becomes a delta function.

$$\Gamma_{x,x+s}(0) = 1 \longrightarrow S_x(w) = \delta(w) \tag{13}$$

$$T(w_1, w_2) = C_3 f(w_1) f^*(w_2) \tag{14}$$

$$I(x) = C_3 \left| \int f(w) A(w) e^{jwx} dw \right|^2 \tag{15}$$

Complete Incoherence

The mutual coherence function for incoherent light is a delta function.

$$\Gamma_{x,x+s}(0) = \delta(s) \longleftrightarrow S_x(w) = 1 \tag{16}$$

Let:

$$\begin{aligned} T_i(w) &\equiv T(w_1, w_2) \\ &= C_3 f(w) * f^*(w) \end{aligned} \tag{17}$$

Equation 17 is the incoherent transfer function.

Let:

$$t_f(x) \longleftrightarrow T_f(w) \tag{18}$$

$T_f(w)$ is the Fourier transform of the light intensity transmission of the transparency.

$$I(x) = \int T_i(w) \; T_f(w) e^{jwx} dw \tag{19}$$

Equation 15 and 19 are the familiar equations for coherent and incoherent imagery respectively. Coherent imagery images the complex electric field transmission while incoherent imagery images light intensity transmission. The simplification in Equation 15 is possible because of the product relationship in Equation 14. In equation 19 the simplification results because $T(w_1, w_2)$ is a function of $(w_1 - w_2)$.

For quasi-coherent light, neither of these simplifications can be made. Thus we return to Equations 11 and 12 and consider a specific example.

We assume, as is usually the case, that $S_x(w)$ is independent of x and let:

$$S(w) = \frac{1}{2\epsilon} P_\epsilon(w). \tag{20}$$

The rectangular pulse function $P_\epsilon(w) \equiv \begin{cases} 1 & \text{for } |w| < \epsilon \\ 0 & \text{for } |w| > \epsilon \end{cases}$

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Equation (20) says that there is equal probability for all frequencies for $|w|<\epsilon$ and frequencies $|w|>\epsilon$ do not exist in the incident light. This is true for an extended, incoherent, but monochromatic source. In addition, let the spatial filter be a simple aperture that passes only the frequencies $|w|<1$.

$$f(w) = P_1(w) \tag{21}$$

The assupmtions of Equations 20 and 21 yield $T(w_1, w_2)$ the "two dimensional transfer function," shown in Figures 2a and 2b. For clarity Figure 2a is shown for $\epsilon<1$ and Figure 2b for $\epsilon>1$. It can be seen that the probability of transmission of two frequencies separated by greater than the passband of the filter is zero. The important thing to note is that if:

$$A(w) = 0 \text{ for } |w|>|1-\epsilon| \tag{22}$$

then for $\epsilon<1$ the imagery is coherent (Equation 15)
and for $\epsilon>1$ the imagery is incoherent (Equation 19).

An almost coherent system can best be approximated by Equation 15 with the filter replaced by the approximation:

$$f_\epsilon(w) = S(w) * f(w) \text{ for } \epsilon << 1 \tag{23}$$

$f_\epsilon(w)$ under the assumptions of Equation 20, 21, and 23 and for $\epsilon<<1$ is shown in Figure 2C. Some more detailed comments on nearly coherent imagery will be made in a future memo.

For the sake of completeness equations 11 and 12 may be extended to two dimensions.

$$I(x,y) = \int T(w_{x1}, w_{y1}, w_{x2}, w_{y2})\, A(w_{x1}, w_{y1})\, A^*(w_{x2}, w_{y2}) \cdot \exp j\left[(w_{x1}-w_{x2})\,x + (w_{y1}-w_{y2})\,y\right] dw_{x1}\, dw_{x2}\, dw_{y1}\, dw_{y2} \tag{24}$$

$$T(w_{x1}, w_{y1}, w_{x2}, w_{y2}) = \int S(w_x', w_y')\, \xi(w_{x1}-w_x', w_{y1}-w_y') \cdot \xi^*(w_{x2}-w_x', w_{y2}-w_y')\, dw_x'\, dw_y' \tag{25}$$

$$S(w_x', w_y') = \int_{x,y} u(x,y,t)\, u^*(x+s_x, y+s_y, t)\, e^{j(s_x w_x' + s_y w_y')}\, ds_x\, ds_y \tag{26}$$

Equations 24 to 26 are direct extensions to two dimensions of the one dimensional model described by Equations 9 to 12.

A future memo will show the images formed by a quasi-coherent optical system for bar and sine wave targets.

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Interference fringes formed from reflections are a problem with coherent imagery. The fraction of light reflected is independent of the degree of coherence. The distribution of the energy is strongly dependent upon the degree of coherence.

Experimentally it was found that many fringes vanished if a "diffuser" was mechanically moved behind the transparency or collimator used to illuminate the transparency. The next section will relate the diffuser to $\Gamma_{x,x+s}(0)$ and $S_x(w)$.

The diffusers were not moved rapidly enough to vary the frequency of the light and thus temporal coherence is preserved. The fringes vanish, because the spatial coherence is reduced. The fringes move in time with the diffuser while the image stands still. The amount the fringes move is related to degree that the coherence is destroyed.

The diffusers used were of two types:

1) A granular translucent diffuser was moved behind the negative during exposure.
2) A glass plate was rocked behind the collimator.

For simplicity the incident light will be assumed to be completely coherent. If the diffuser undergoes linear motion and the glass plate periodic motion Equation 10 takes the following forms. For uniform illumination $\Gamma_{x,x+s}(0)$ is not a function of x in either case.

For the diffuser:

$$\Gamma_s(0) = \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} U_1(x)\, U_1(x+s)\, dx \tag{27}$$

$\Gamma_s(0)$ is the auto correlation function of $U_1(x)$, the phase variation, in the material.

For the rocking plate:

$$U(x,t) = \exp j\, \frac{2\pi x}{\lambda}\, \phi(t) \quad \text{for } |\phi(t)| \ll 1 \tag{28}$$

$\phi(t)$ is the angle through which the plane wave is rotated by the periodic motion of the rocking plate. This case is best analyzed by considering Equation 9.

Equation 9 may be reduced because $\phi(t)$ is periodic.[2]

$$S(w) = C_4 \left[\frac{d\phi(t_1)}{dt_1} \right]^{-1} \tag{29}$$

Where t_1 is the solution to the equation:

$$\phi(t_1) = w \tag{30}$$

As an example, if the rocking is linear for one period.

$\phi(t)$ for $|t| \le 1$ (31)

Then $S(w) = \frac{1}{2\xi} P_{\xi}(w)$ (32)

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Equation 32 is the same as the example considered for Equation 20, thus a linearly rocked plate is exactly the same as an extended monochromatic source that is spatially incoherent. This equivalence may be explained by imagining the plate in several positions at once in an unrelated manner. This is the description of a monochromatic source that emits many unrelated plane waves simultaneously. The fringes vanish because as the plate rocks the rays pass through different portions of the optics causing the reflected images to move. This causes the fringes to move with time and the average intensity of the fringes is zero.

This memo has described the effect of partial spatial coherence on imagery. A following memo will consider the effect on imagery for several light sources as a function of intensity in the image plane. This memo will show that reflection fringes can be eliminated and high intensity achieved while maintaining a quasi-coherent imagery that is described by Equation 20 to 23 with $\epsilon \ll 1$ The quasi-coherent imagery will be almost indistinguishable from completely coherent imagery.

REFERENCES

1. M. Born, E. Wolf, "Principles of Optics," p. 490-496, 500-502, Pergamon Press, N.Y. 1959.

2. A. Papoulis, "The Fourier Integral and Its Applications," Chapter 1-3, 12, Appendix 1, McGraw-Hill, N.Y., 1962.

3. W. Davenport W Root, "Random Signals and Noise," p. 32-38, 59-61, 66-70, 101-107, McGraw-Hill, N.Y., 1958.

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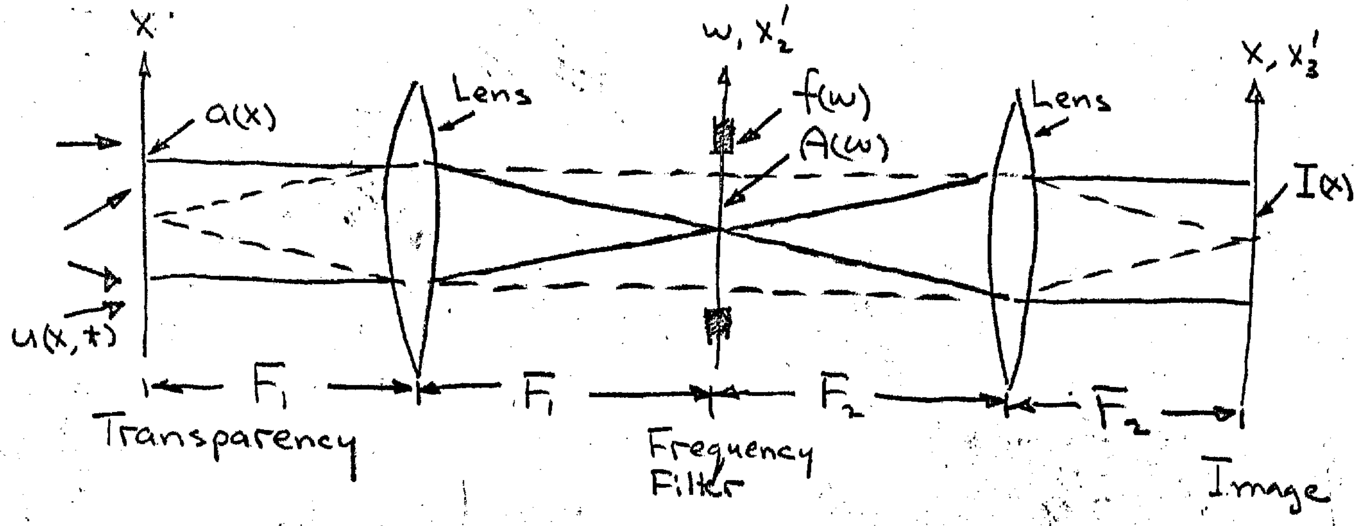

Fig. 1. Optical System Model

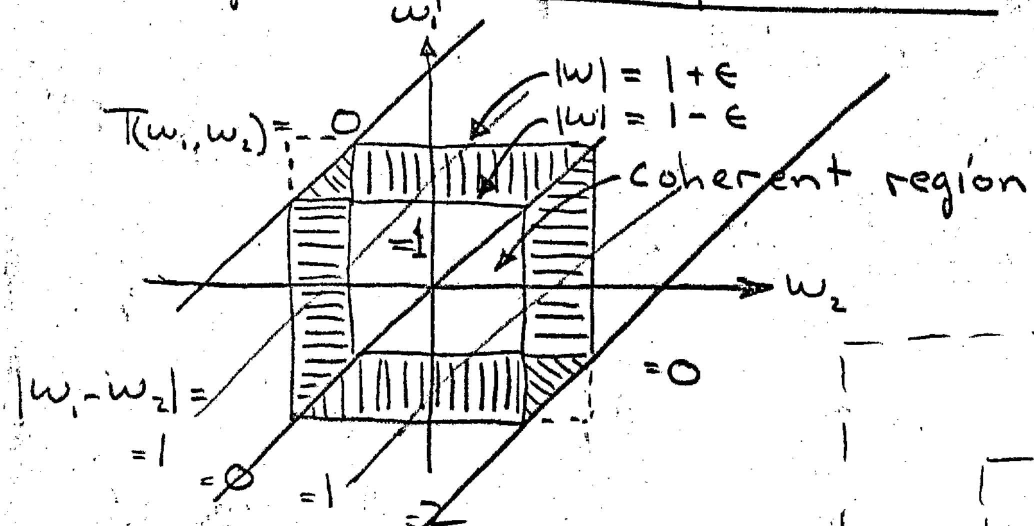

Fig. 2a. $\epsilon < 1$

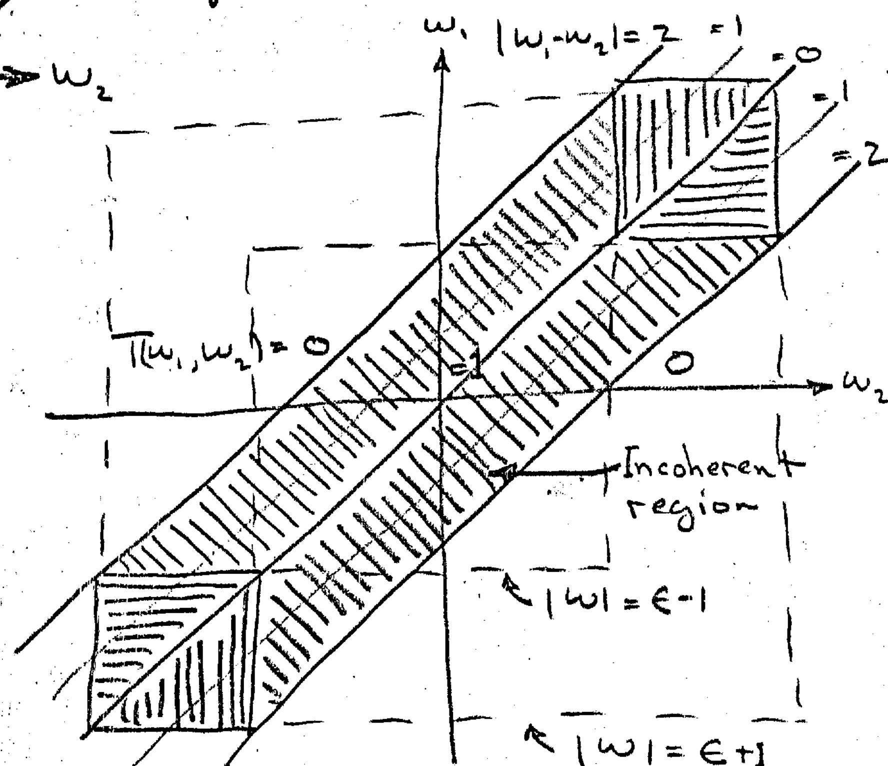

Fig. 2b. $\epsilon > 1$

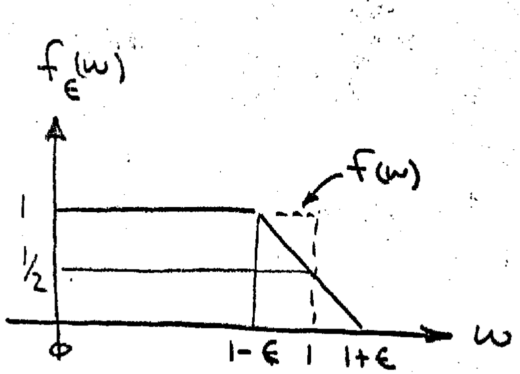

Fig. 2c. $\epsilon \ll 1$

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Fig 2 Quasi-Coherent Transfer Functions

The closely spaced lines follow the slope of $T(\omega_1,\omega_2)$.

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COHERENT LIGHT ENLARGER AND SPATIAL FILTER
6th Progress Report: 16 Nov. to 11 Dec. '63

Progress has continued at a somewhat more rapid pace and we are still expecting to meet the original delivery date. Items of interest:

1. All lenses and cells have been released for fabrication with no change of delivery date as compared to the original pert chart.

2. Work on the electrical schematic and packaging has begun and has led to a possible improvement of the film transport method. This work is being intensively pursued with an eye toward making film transport simplier to operate.

3. A comprehensive series of tests using both laser and incandescent light sources was completed and it is expected that the results of these tests will be shown to the customer by the end of the month. We have not as yet determined the transfer function of the enlarger. Experimentation is now concentrated on the electronic focus indicator which has been loaned to us by Logetronics. This experiment should be completed by the end of this week.

4. Due to availability of manpower and machines in our own Model Shop, we have started to manufacture lens mounts and other equipment here

STAT [redacted] This will result in lower costs and more reliable delivery dates.

5. Our facilities now include the breadboard enlarger room and assembly room for the prototype and a dark room, all in a nonclassified area.

Activity for the coming month should include completion of all mechanical drawings, determination of transfer function, fabrication of additional spatial filters, continued effort on theoretical studies and a trip to the customer to review results.

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— No financial report!